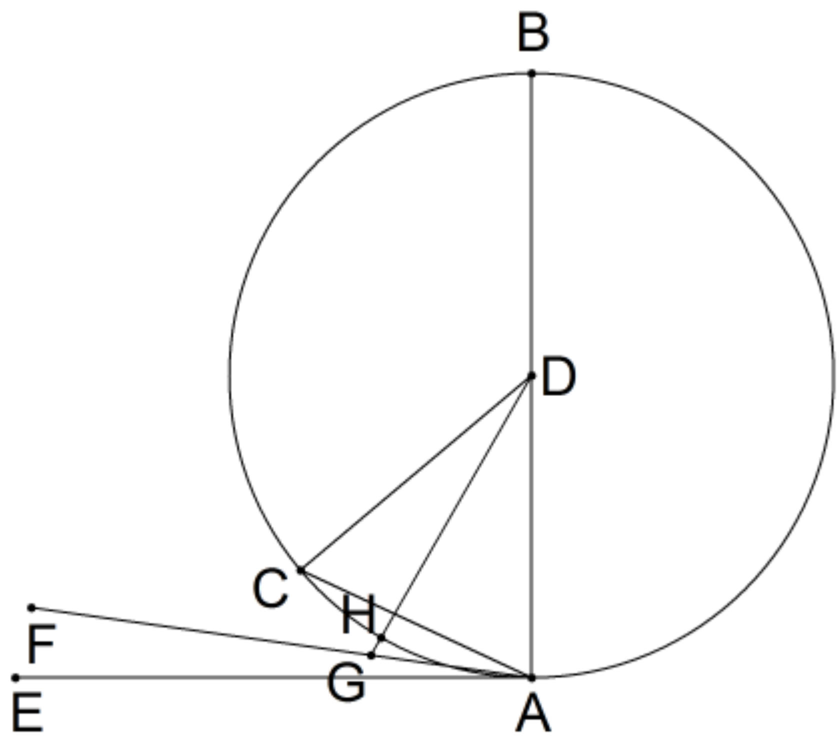


Paraphrase of Euclid Book 3 Proposition 16

a) A straight line (AE) drawn perpendicular to the diameter of a circle will fall outside the circle.
b) Another straight line cannot be drawn in the space between AE and the circumference.
c) The angle of the semicircle [between the diameter and the circumference] is greater than any acute rectilinear angle, and the remaining angle [formed by the semicircle and the tangent line AE] is less.

Proof of (a)
If the line does not fall outside the circle, it will intersect the circle at point A and another point C (line AC on the diagram). Let there be a line DC from the center to the second point.
Since AD and DC are equal in length (both are radii of the circle), then their opposite angles ACD and CAD must be equal (see Proposition 1.5).
But the angle CAD must be a right angle given the premise, meaning that ACD must also be right, which is impossible for the triangle ACD (Proposition 1.17).
Therefore, the line falls outside the circle.

Proof of (b)

If there can be another line drawn between AE and the circle, draw it as AF and have a line from the center (line DG) perpendicular to it, intersecting the circle at point H and the line at point G.

Since angle AGD is a right angle, and angle DAG is less than a right angle, AD must be greater than DG (Proposition 1.19).

But AD equals DH (both are radii of the circle). Therefore, DH must be greater than DG, which is impossible if line AF is outside the circle.

Proof of (c)

If an angle exists greater than the angle formed by the diameter AB and the circumference CHA, or less than the angle formed by the circumference CHA and the tangent line AE, then there must exist a line between the circle and AE (AF) such that it will form an angle FAB that is greater than the angle of the diameter and the circumference, and an angle FAE that is smaller than the angle of the tangent line and the circumference.

But we have just proved that no such line can exist; therefore, an acute rectilinear angle greater than the angle of AB and CHA, or smaller than the angle of AE and CHA, cannot exist.

Corollary:

From this it is apparent that a line drawn at right angles to the diameter of the circle from its extremity touches the circle (and that the line touches the circle at a single point, since it was also shown that a line meeting the circle at two points falls within it [Proposition 3.2]). Q.E.D.

See also: http://www.claymath.org/library/historical/euclid/

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